Direct Numerical Simulations of Multiphase Flows-9

Exercises

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DNS of Multiphase Flows

More advanced exercises
Examine the motion of two or more bubbles
Add the heat equation and variable surface tension and examine thermocapillary migration of a drop
Solve for the electric field between two plates and add electrostatic forces to the interface

DNS of Multiphase Flows

Thermocapillary migration


https://dl.dropbox.com/u/7336270/NasMuradogluTrygg06.pdf
https://dl.dropbox.com/u/7336270/NasTryggvason03.pdf

DNS of Multiphase Flows

Variable Surface Tension

Variable surface tension can be included in a very straightforward way

In two-dimension, the rate of change of surface tension generally results in both a normal and a tangential component

\[
\frac{\partial \sigma}{\partial t} = \kappa \mathbf{n} + \left( \frac{\partial \sigma}{\partial t} \right) \mathbf{n}
\]

Similarly, in three-dimension:

\[
\mathbf{F} = \int \frac{\partial \sigma}{\partial t} \mathbf{n} \, ds
\]

The force on an element is:

\[
\mathbf{F} = \int \frac{\partial \sigma}{\partial t} \mathbf{n} \, ds
= (\mathbf{a} \mathbf{n}) - (\mathbf{a} \mathbf{n})
\]
The temperature is found by solving the energy equation

\[ \rho c_p \frac{\partial T}{\partial t} + \rho c_p \nabla \cdot (\dot{T}u) = \nabla \cdot k \nabla T \]

where we have assumed that the fluid is incompressible and that viscous heating can be neglected. This equation is solved on a fixed grid by an explicit second order method in the same way as the momentum equation. The temperature on the surface of the bubble or drop is found by interpolating it from the grid and surface tension is found by

\[ \sigma = \sigma_o - \beta (T - T_o) \]

Here, \( \beta > 0 \), since surface tension generally is reduced with increasing temperature.

**Thermocapillary migration—I**

Thermocapillary motion of many two-dimensional bubbles. The top wall is hot and the bottom wall is cold. Initially the bubbles are placed near the cold wall. As they rise, they form horizontal layers. These layers, however, block the flow and become unstable to let the fluid flow toward the cold wall as the bubbles move toward the hot one.

**Thermocapillary migration—II**

Thermocapillary motion of two three-dimensional bubbles. The top wall is hot and the bottom wall is cold. Initially the bubbles are placed near the cold wall. As they rise, the bubbles line up perpendicular to the temperature gradient.

**Thermocapillary migration—III**

Electrohydrodynamics


https://dl.dropbox.com/u/7336270/Fernandez05.pdf

**Electrohydrodynamics**

Momentum (conservative form, variable density and viscosity)

\[ \frac{\partial u}{\partial t} + \rho \dot{u} \cdot \nabla \dot{u} = -\nabla p + \frac{\sigma}{\kappa} n \delta x - x f \nabla \cdot u = 0 \]

Electric force

\[ \sigma \nabla \cdot E = 0 \]

Mass conservation (incompressible flows)

\[ \nabla \cdot \dot{u} = 0 \]

Electrostatic motion—I

For fluids with small but finite conductivity, Taylor and Melcher (1969) proposed the “leaky dielectric” model. This model allows both normal and tangential electrostatic forces on a two fluid interface.
### Electrostatic motion—II

The electric field is obtained from the electric potential by

\[ \nabla \cdot \sigma \nabla \phi = 0 \]

where

\[ E = \nabla \phi \]

The force on the fluid is then found by:

\[ f = qE - \frac{1}{2}(E \cdot E)\nabla \epsilon \]

where the charge accumulation is

\[ q = \nabla \cdot \epsilon \vec{E} \]

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### Interaction of Two Drop

The motion of two oblate drops in a quiescent flow. The drops align with the electric field and attract each other. The drops are also attracted to the wall.